

Multigenerative Grammar Systems (MGS)

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- This presentation is based on publication:

Meduna, A, Lukáš, R: Multigenerative Grammar systems,
Schedae Informaticae

From Grammar To MGS

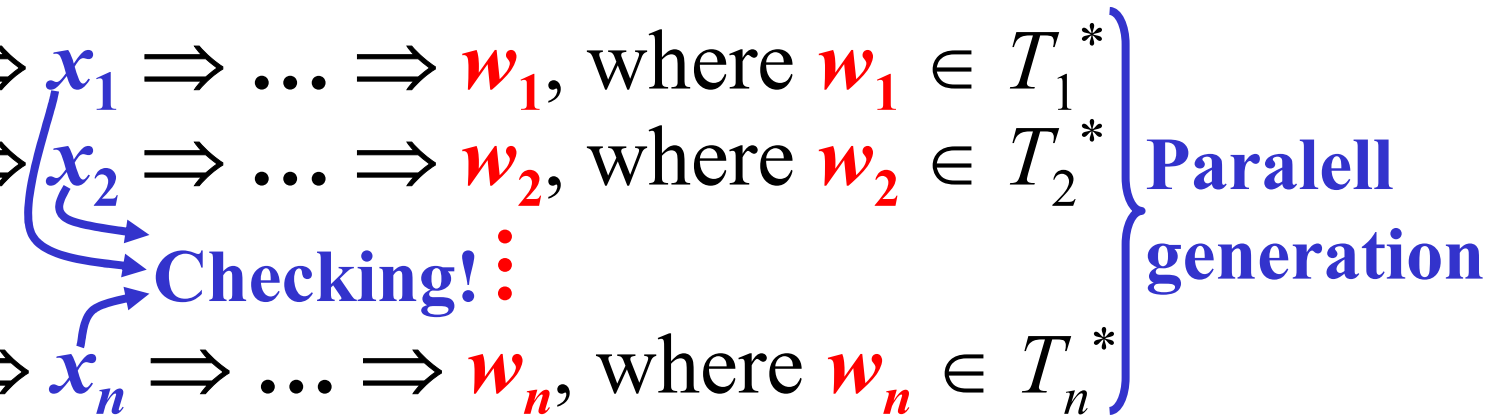
Grammar: $G = (N, T, P, \mathbf{S})$

$\mathbf{S} \Rightarrow \dots \Rightarrow \dots \Rightarrow \mathbf{w}$, where $\mathbf{w} \in T^*$

Grammar system: $\Gamma = (G_1, G_2, \dots, G_n, Q)$, where

- $G_i = (N_i, T_i, P_i, \mathbf{S}_i)$ is a CFG for all $i = 1 \dots n$
- Q = a set of rules, which „check“ generation.

$\mathbf{S}_1 \Rightarrow \mathbf{x}_1 \Rightarrow \dots \Rightarrow \mathbf{w}_1$, where $\mathbf{w}_1 \in T_1^*$
 $\mathbf{S}_2 \Rightarrow \mathbf{x}_2 \Rightarrow \dots \Rightarrow \mathbf{w}_2$, where $\mathbf{w}_2 \in T_2^*$
 \vdots
 $\mathbf{S}_n \Rightarrow \mathbf{x}_n \Rightarrow \dots \Rightarrow \mathbf{w}_n$, where $\mathbf{w}_n \in T_n^*$

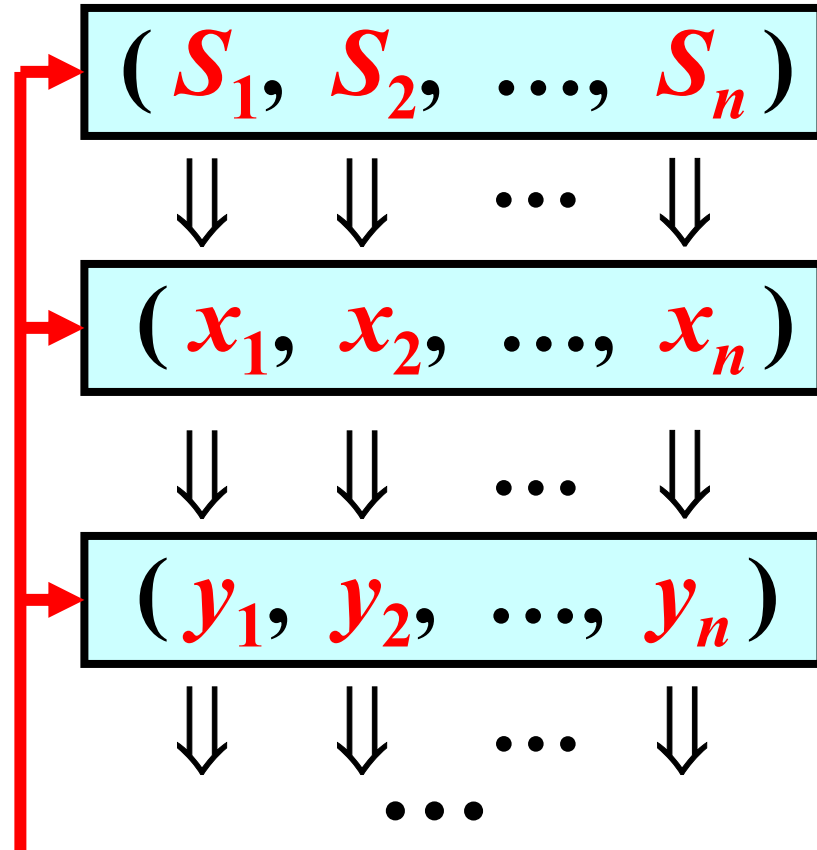

Paralell generation

Checking!

$(\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_n)$ = Generated multistring in Γ .

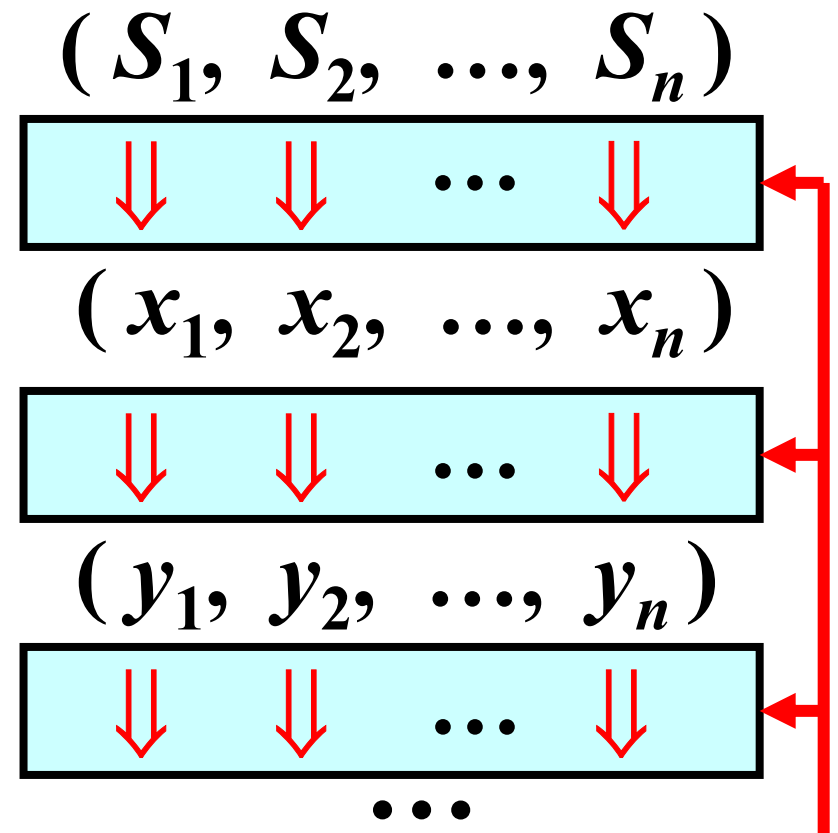
Checking: Two Approaches

1) Symbols Checking:



Checking of multiforms

2) Rules Checking:



Checking of derivations

Nonterminal-synchronized GS

Definition: An n -multigenerative nonterminal-synchronized grammar system (n -MGN) is $n+1$ tuple

$$\Gamma = (G_1, G_2, \dots, G_n, Q), \text{ where}$$

- $G_i = (N_i, T_i, P_i, S_i)$ is a CFG for all $i = 1 \dots n$
- Q is a finite set of n -tuples of the form (A_1, A_2, \dots, A_n) , where $A_i \in N_i$ for all $i = 1 \dots n$

Example:

$\Gamma = (G_1, G_2, \{(S_1, S_2), (A_1, A_2)\})$, where:

$$G_1 = (\{S_1, A_1\}, \{a, b, c\}, R_1, S_1);$$

$$R_1 = \{S_1 \rightarrow aS_1, S_1 \rightarrow aA_1, A_1 \rightarrow bA_1c, A_1 \rightarrow bc\}$$

$$G_2 = (\{S_2, A_2\}, \{d\}, R_2, S_2);$$

$$R_2 = \{S_2 \rightarrow S_2A_2, S_2 \rightarrow A_2, A_2 \rightarrow d\}$$

Direct Derivation Step

Definition: Let $\Gamma = (G_1, G_2, \dots, G_n, Q)$ be a n-MGN. Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, $A_i \rightarrow x_i \in P_i$ for all $i = 1..n$. Then, $(u_1 A_1 v_1, u_2 A_2 v_2, \dots, u_n A_n v_n) \Rightarrow (u_1 x_1 v_1, u_2 x_2 v_2, \dots, u_n x_n v_n)$ if $(A_1, A_2, \dots, A_n) \in Q$.

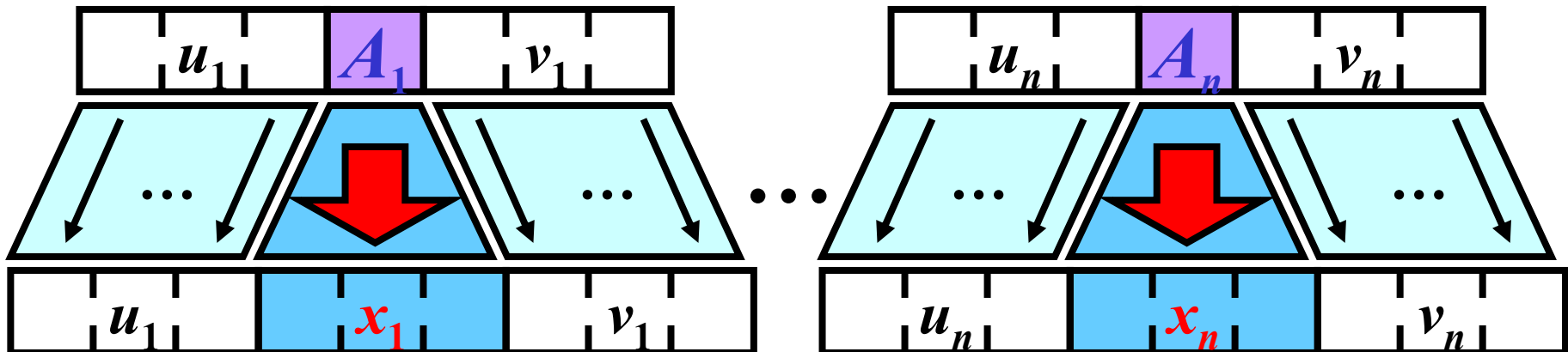
Note: \Rightarrow^+ ... transitive closure of \Rightarrow

\Rightarrow^* ... reflexive and transitive closure of \Rightarrow

Illustration: $(A_1, \dots, A_n) \in Q$

Rule: $A_1 \rightarrow x_1 \in P_1$

Rule: $A_n \rightarrow x_n \in P_n$

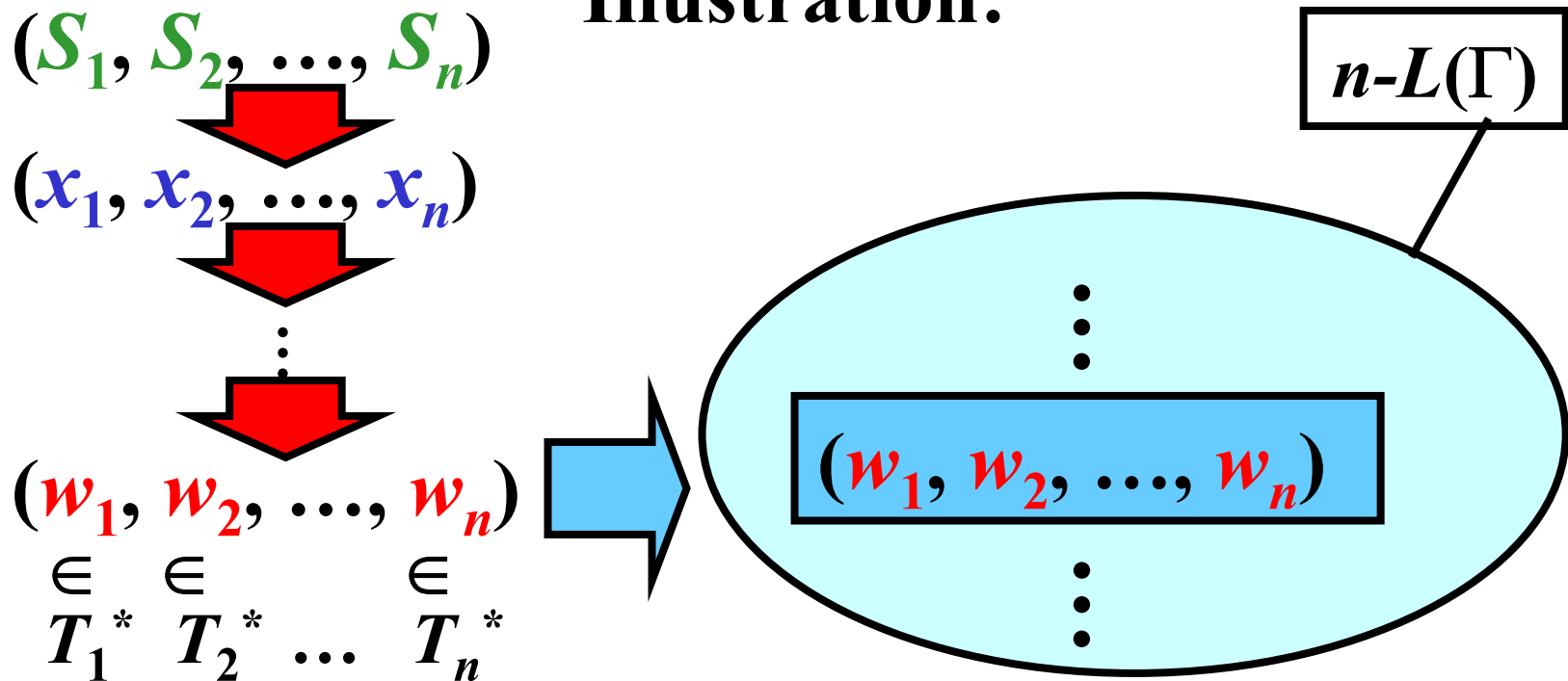


n -Language

Definition: Let $\Gamma = (G_1, G_2, \dots, G_n, Q)$ be a n -MGN. The n -Language of Γ , $n-L(\Gamma)$, is defined as:

$$n-L(\Gamma) = \{(w_1, w_2, \dots, w_n) : (S_1, S_2, \dots, S_n) \Rightarrow^* (w_1, w_2, \dots, w_n), \\ w_i \in T_i^* \text{ for all } i = 1, \dots, n\}$$

Illustration:

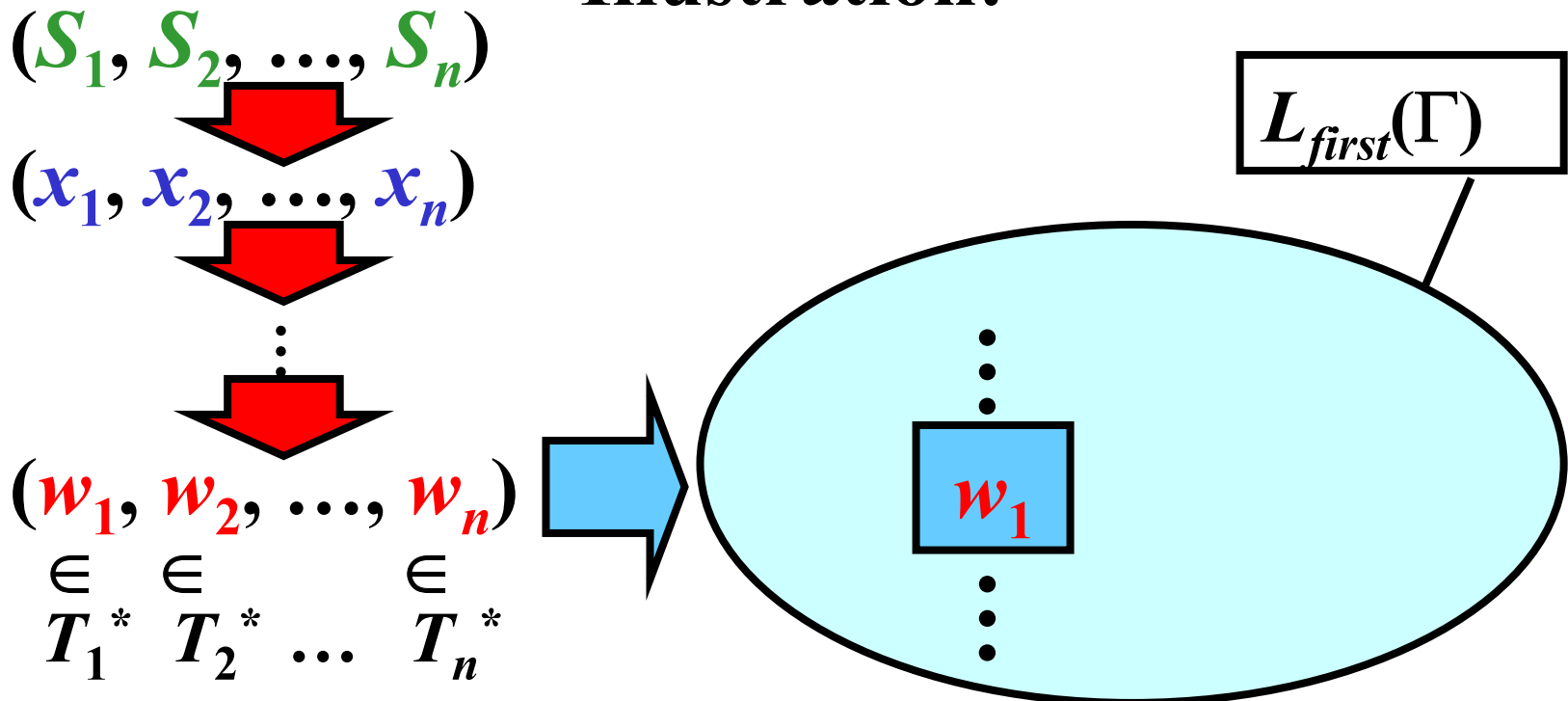


Generated Language in *First Mode*

Definition: The *language generated by Γ in the first mode*, $L_{first}(\Gamma)$, is defined as:

$$L_{first}(\Gamma) = \{w_1 : (w_1, w_2, \dots, w_n) \in n-L(\Gamma)\}$$

Illustration:

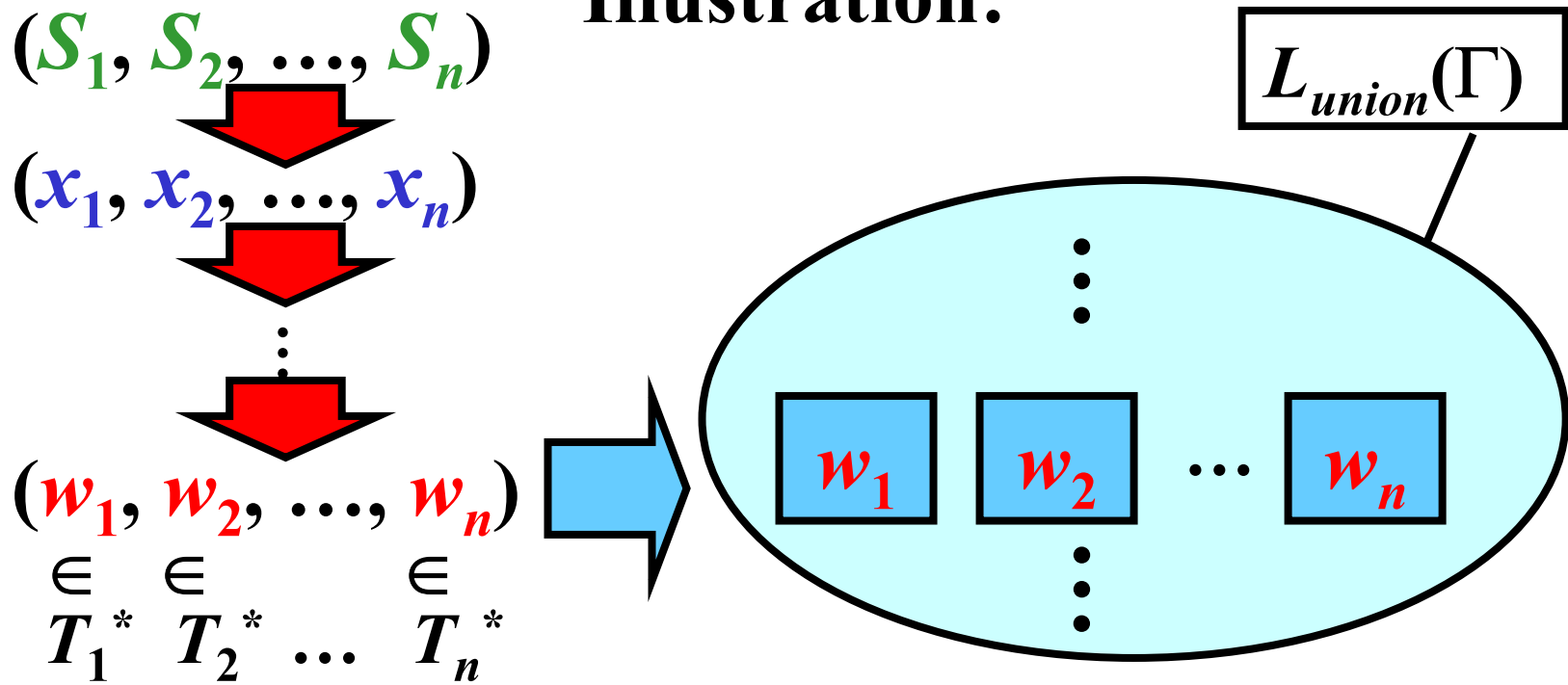


Generated Language in *Union Mode*

Definition: The *language generated by Γ in the union mode*, $L_{union}(\Gamma)$, is defined as:

$$L_{union}(\Gamma) = \{w: (w_1, w_2, \dots, w_n) \in n-L(\Gamma), \\ w \in \{w_i: i = 1, \dots, n\}\}$$

Illustration:

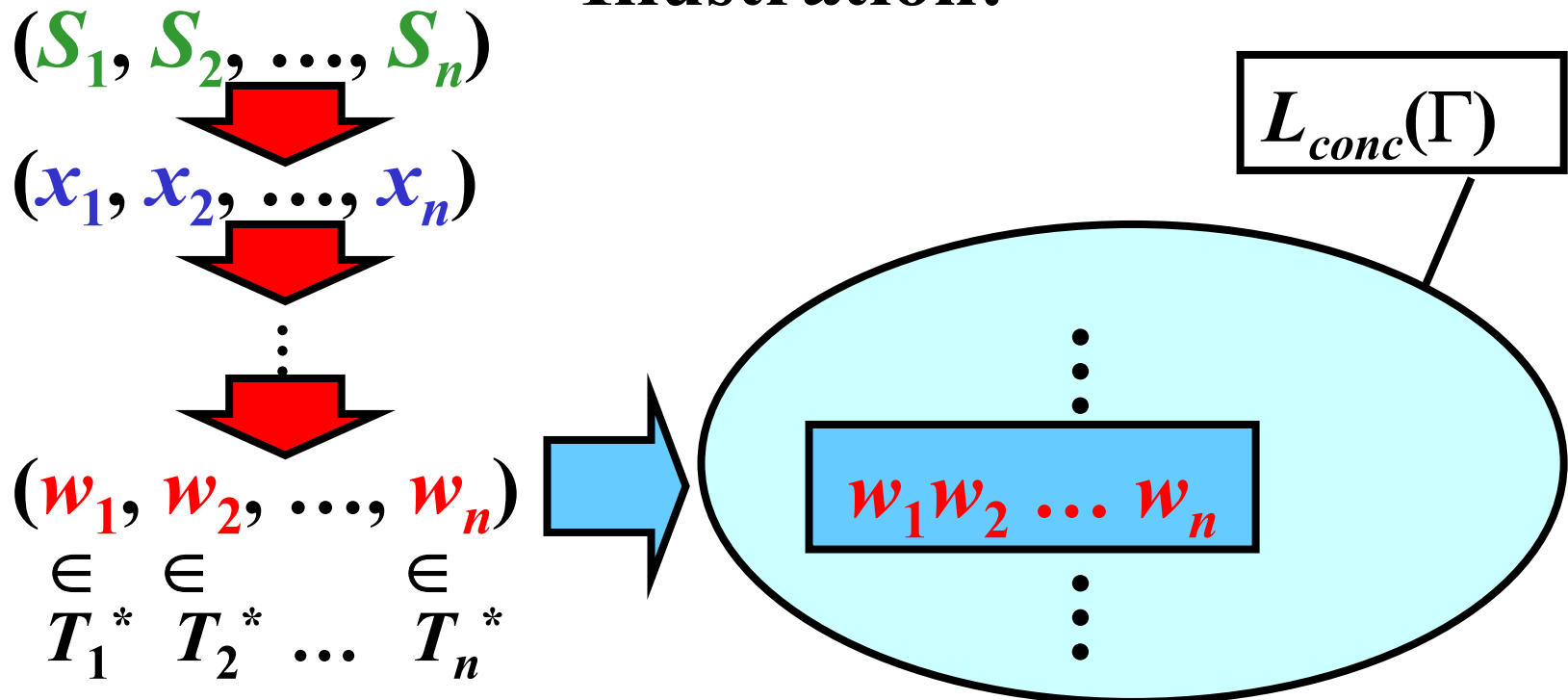


Generated Language in *Conc. Mode*

Definition: The *language generated by Γ in the concatenation mode*, $L_{conc}(\Gamma)$, is defined as:

$$L_{conc}(\Gamma) = \{w_1 w_2 \dots w_n : (w_1, w_2, \dots, w_n) \in n-L(\Gamma)\}$$

Illustration:



n-MGN: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

• $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\},$

$T_1 = \{a, b, c\},$

$R_1 = \{S_1 \rightarrow aS_1, S_1 \rightarrow aA_1,$
 $A_1 \rightarrow bA_1c, A_1 \rightarrow bc\}$

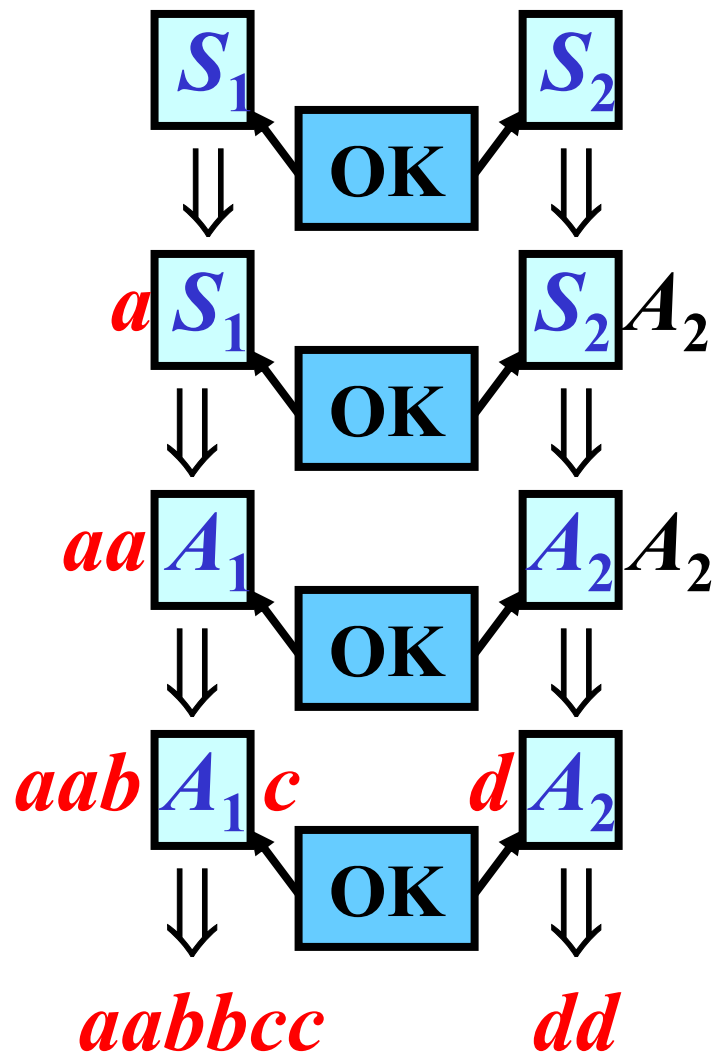
• $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2, A_2\},$

$T_2 = \{d\},$

$R_2 = \{S_2 \rightarrow S_2A_2, S_2 \rightarrow A_2,$
 $A_2 \rightarrow d\}$

• $Q = \{(S_1, S_2), (A_1, A_2)\}$



n-MGN: Example 2/2

$\Gamma = (G_1, G_2, Q)$, where:

- $G_1 = (N_1, T_1, R_1, S_1)$
 $N_1 = \{\textcolor{blue}{S}_1, \textcolor{blue}{A}_1\},$
 $T_1 = \{\textcolor{red}{a}, \textcolor{red}{b}, \textcolor{red}{c}\},$
 $R_1 = \{\textcolor{blue}{S}_1 \rightarrow \textcolor{red}{a}\textcolor{blue}{S}_1, \textcolor{blue}{S}_1 \rightarrow \textcolor{red}{a}\textcolor{blue}{A}_1,$
 $\textcolor{blue}{A}_1 \rightarrow \textcolor{red}{b}\textcolor{blue}{A}_1\textcolor{red}{c}, \textcolor{blue}{A}_1 \rightarrow \textcolor{red}{b}\textcolor{red}{c}\}$

- $G_2 = (N_2, T_2, R_2, S_2)$
 $N_2 = \{\textcolor{blue}{S}_2, \textcolor{blue}{A}_2\},$
 $T_2 = \{\textcolor{red}{d}\},$
 $R_2 = \{\textcolor{blue}{S}_2 \rightarrow \textcolor{blue}{S}_2\textcolor{blue}{A}_2, \textcolor{blue}{S}_2 \rightarrow \textcolor{blue}{A}_2,$
 $\textcolor{blue}{A}_2 \rightarrow \textcolor{red}{d}\}$

- $Q = \{(\textcolor{blue}{S}_1, \textcolor{blue}{S}_2), (\textcolor{blue}{A}_1, \textcolor{blue}{A}_2)\}$

Note:

$$2-L(\Gamma) = \{(\textcolor{red}{a}^n \textcolor{red}{b}^n \textcolor{red}{c}^n, \textcolor{red}{d}^n) : n \geq 1\}$$

$$L(\Gamma)_{union} = \{\textcolor{red}{a}^n \textcolor{red}{b}^n \textcolor{red}{c}^n : n \geq 1\} \cup \{\textcolor{red}{d}^n : n \geq 1\}$$

$$L(\Gamma)_{conc} = \{\textcolor{red}{a}^n \textcolor{red}{b}^n \textcolor{red}{c}^n \textcolor{red}{d}^n : n \geq 1\}$$

$$L(\Gamma)_{first} = \{\textcolor{red}{a}^n \textcolor{red}{b}^n \textcolor{red}{c}^n : n \geq 1\}$$

Rule-synchronized GS

Definition: An n -multigenerative rule-synchronized grammar system (n -MGR) is $n+1$ tuple

$$\Gamma = (G_1, G_2, \dots, G_n, Q), \text{ where}$$

- $G_i = (N_i, T_i, P_i, S_i)$ is a CFG for all $i = 1 \dots n$
- Q is a finite set of n -tuples of the form (p_1, p_2, \dots, p_n) , where $p_i \in P_i$ for all $i = 1 \dots n$

Example:

$\Gamma = (G_1, G_2, \{(1, 1), (2, 2), (3, 3), (4, 3)\})$, where:

$G_1 = (\{S_1, A_1\}, \{a, b, c\}, R_1, S_1);$

$$R_1 = \{1: S_1 \rightarrow aS_1, \quad 2: S_1 \rightarrow aA_1, \\ 3: A_1 \rightarrow bA_1c, \quad 4: A_1 \rightarrow bc\}$$

$G_2 = (\{S_2\}, \{d\}, R_2, S_2);$

$$R_2 = \{1: S_2 \rightarrow S_2S_2, \quad 2: S_2 \rightarrow S_2, \quad 3: S_2 \rightarrow d\}$$

Direct Derivation Step

Definition: Let $\Gamma = (G_1, G_2, \dots, G_n, Q)$ be a n-MGR. Let $u_i \in T_1^*$, $v_i \in (N_i \cup T_i)^*$, $p_i: A_i \rightarrow x_i \in P_i$ for all $i = 1..n$. Then, $(u_1 A_1 v_1, u_2 A_2 v_2, \dots, u_n A_n v_n) \Rightarrow (u_1 x_1 v_1, u_2 x_2 v_2, \dots, u_n x_n v_n)$ if $(p_1, p_2, \dots, p_n) \in Q$.

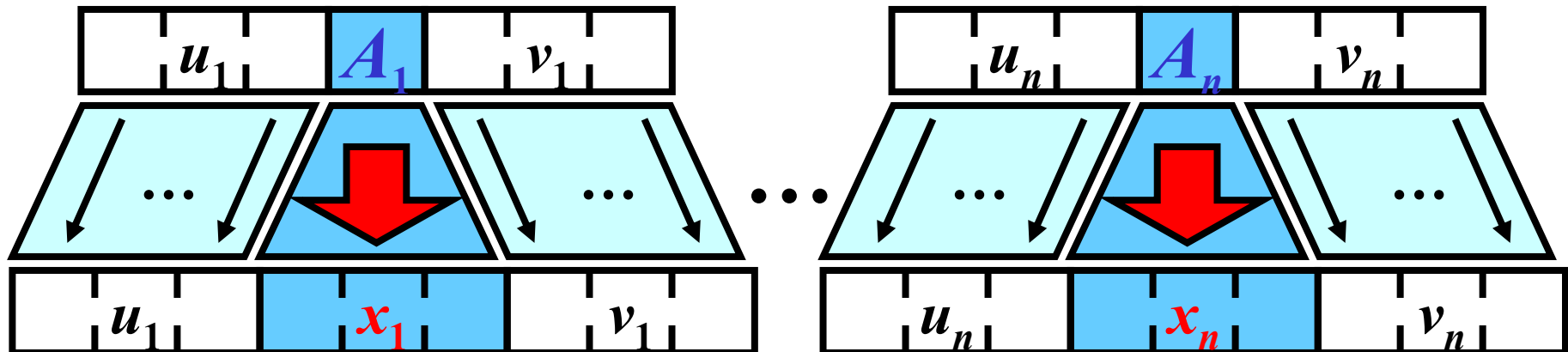
Note: \Rightarrow^+ ... transitive closure of \Rightarrow

\Rightarrow^* ... reflexive and transitive closure of \Rightarrow

Illustration: $(p_1, \dots, p_n) \in Q$

Rule: $p_1: A_1 \rightarrow x_1 \in P_1$

Rule: $p_n: A_n \rightarrow x_n \in P_n$



Generated Multistrings & Language

Definition: An n -Language for n-MGR is defined analogically as the n -Language for n-MGN.

Definition: A language generated by n-MGR in the X mode, for each $X \in \{union, conc, first\}$, is defined analogically as a language generated by n-MGN in the X mode.

n-MGR: Example 1/2

$\Gamma = (G_1, G_2, Q)$, where:

• $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\},$

$T_1 = \{a, b, c\},$

$R_1 = \{1: S_1 \rightarrow aS_1, 2: S_1 \rightarrow aA_1,$
 $3: A_1 \rightarrow bA_1c, 4: A_1 \rightarrow bc\}$

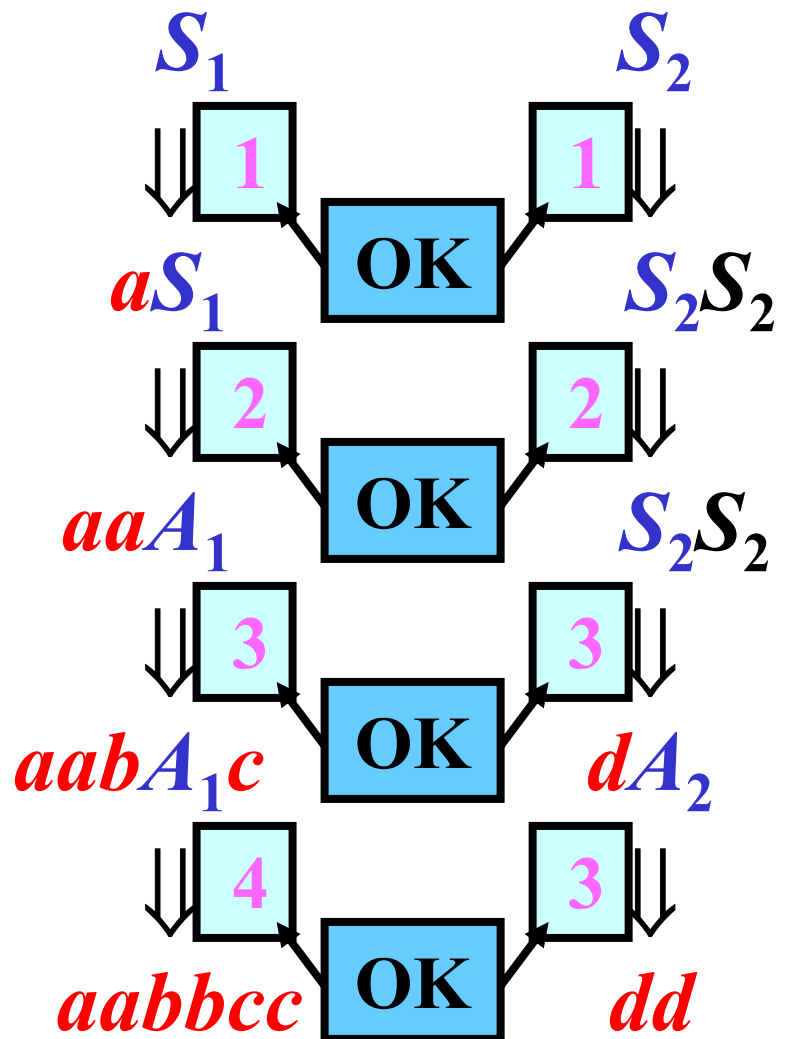
• $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\},$

$T_2 = \{d\},$

$R_2 = \{1: S_2 \rightarrow S_2S_2, 2: S_2 \rightarrow S_2,$
 $3: S_2 \rightarrow d\}$

• $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$



n-MGR: Example 2/2

$\Gamma = (G_1, G_2, Q)$, where:

• $G_1 = (N_1, T_1, R_1, S_1)$

$N_1 = \{S_1, A_1\},$

$T_1 = \{a, b, c\},$

$R_1 = \{1: S_1 \rightarrow aS_1, 2: S_1 \rightarrow aA_1,$
 $3: A_1 \rightarrow bA_1c, 4: A_1 \rightarrow bc\}$

• $G_2 = (N_2, T_2, R_2, S_2)$

$N_2 = \{S_2\},$

$T_2 = \{d\},$

$R_2 = \{1: S_2 \rightarrow S_2S_2, 2: S_2 \rightarrow S_2,$
 $3: S_2 \rightarrow d\}$

• $Q = \{(1, 1), (2, 2), (3, 3), (4, 3)\}$

Note:

$$n-L(\Gamma) = \{(a^n b^n c^n, d^n) : n \geq 1\}$$

$$L(\Gamma)_{union} = \{a^n b^n c^n : n \geq 1\} \cup \{d^n : n \geq 1\}$$

$$L(\Gamma)_{conc} = \{a^n b^n c^n d^n : n \geq 1\}$$

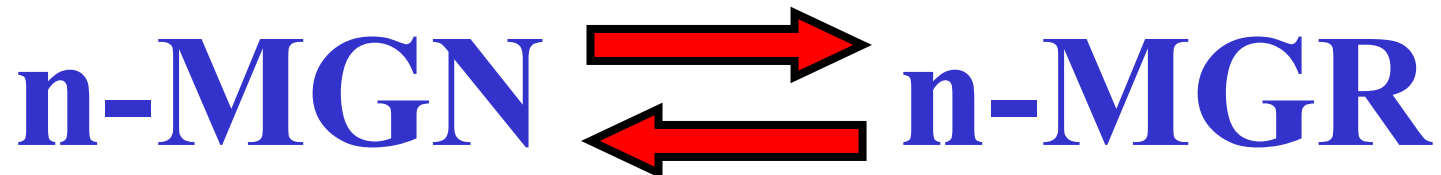
$$L(\Gamma)_{first} = \{a^n b^n c^n : n \geq 1\}$$

Results: Conversions Between MGSs

• **Theorem:** There exist the algorithm, which convets any n-MGN to equivalent n-MGR in the X mode, where $X = \{union, conc, first\}$

• **Theorem:** There exist the algorithm, which convets any n-MGR to equivalent n-MGN in the X mode, where $X = \{union, conc, first\}$

Illustration:

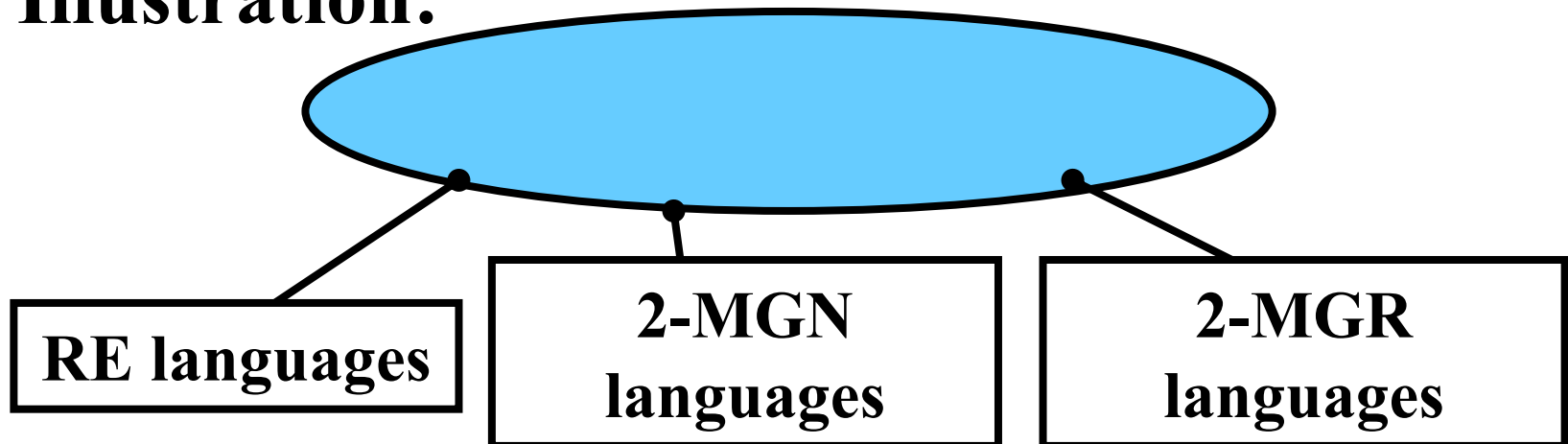


Results: Power of MGSs

• **Theorem:** Let $L(\text{n-MGN}_X)$ and $L(\text{n-MGR}_X)$ denote the language families defined by n-MGN in the X mode and n-MGR in the X mode resp., where $X = \{\text{union}, \text{conc}, \text{first}\}$. Then,

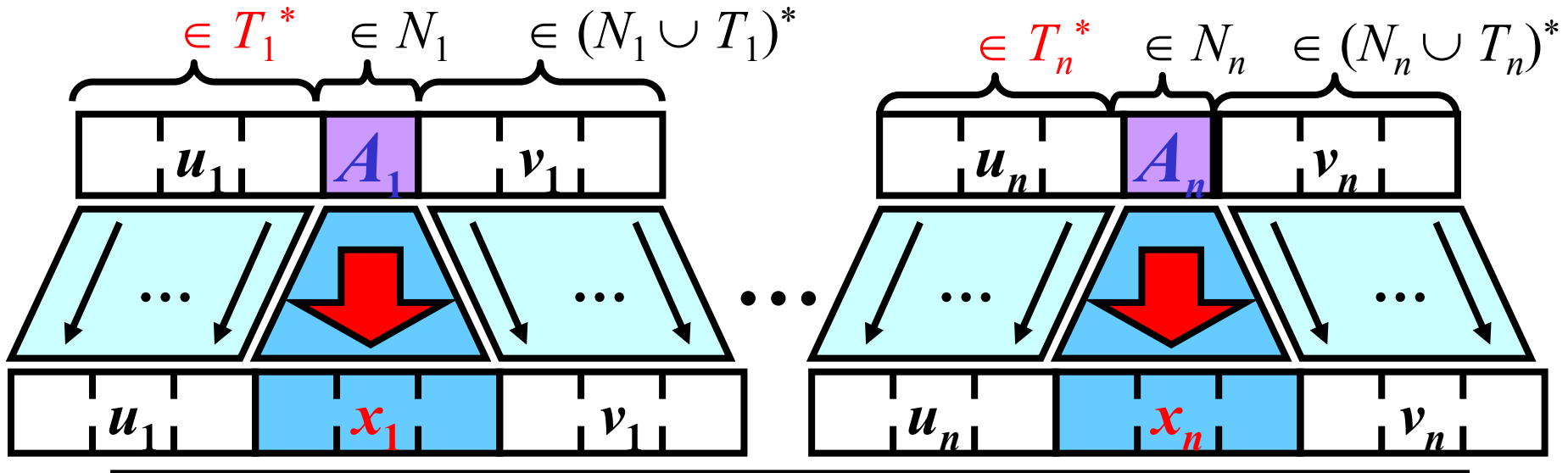
$$L(\text{RE}) = L(2\text{-MGR}_X) = L(2\text{-MGN}_X)$$

Illustration:

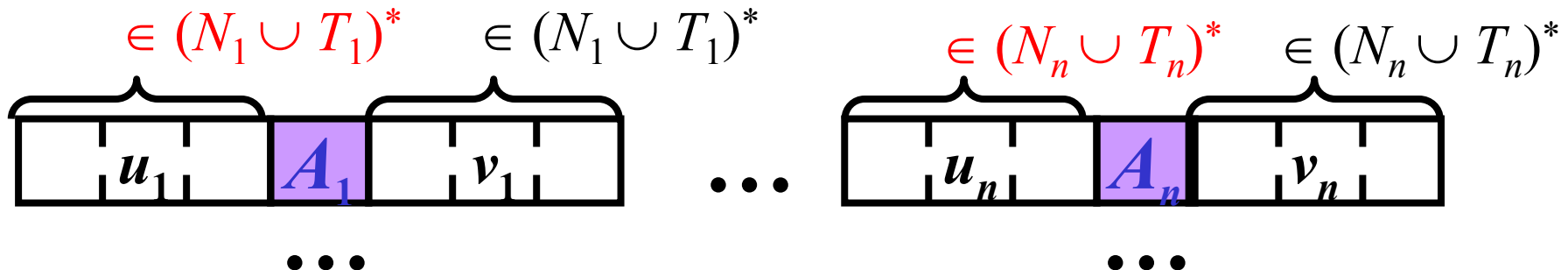


General n-MGNs and n-MGRs

- There is used only the leftmost derivation in n-MGNs/n-MGRs:



- There is used general derivation in General n-MGNs/n-MGRs:



Matrix Grammar

Definition: A *matrix grammar* (MG) is a pair

$$H = (\mathbf{G}, \mathbf{M}), \text{ where:}$$

- $\mathbf{G} = (\mathbf{N}, \mathbf{T}, \mathbf{P}, \mathbf{S})$ is a CFG
- \mathbf{M} is a finite language over \mathbf{P} ($\mathbf{M} \subseteq \mathbf{P}^*$)

Example:

$$H = (G, M), \text{ where:}$$

- $G = (\{S, A, B\}, \{a, b, c\}, P, S);$
 $P = \{1: S \rightarrow AB, 2: A \rightarrow aA, 3: B \rightarrow bBc,$
 $4: A \rightarrow a, 5: B \rightarrow bc\}$
- $M = \{1, 23, 45\}$

Matrix Grammar: Example

$H = (G, M)$, where:

- $G = (\{S, A, B\}, \{a, b, c\}, P, S)$;
 $P = \{1: S \rightarrow AB, 2: A \rightarrow aA, 3: B \rightarrow bBc, \\ 4: A \rightarrow a, 5: B \rightarrow bc\}$
 - $M = \{1, 23, 45\}$
-

$$\underline{S} \Rightarrow \underline{AB} [1] \Rightarrow abc [45]$$

$$\underline{S} \Rightarrow \underline{AB} [1] \Rightarrow a\underline{A}b\underline{B}c [23] \Rightarrow aabbcc [45]$$

...

$$L(H) = \{a^n b^n c^n : n \geq 1\}$$

MG and General n-MGNs & n-MGRs: Conversions

• **Theorem:** For every general n-MGN & n-MGR in the X mode, where $X = \{union, conc, first\}$ there exists an equivalent matrix grammar.

• **Theorem:** For every matrix grammar there exists an equivalent n-MGN & n-MGR in the X mode, where $X = \{union, conc, first\}$

Illustration:

General n-MGN \Rightarrow MG \Rightarrow General 2-MGN
 General n-MGR \Rightarrow MG \Rightarrow General 2-MGR

Results: Power of General MGSs

• **Theorem:** Let $L(\text{n-GMGN}_X)$ and $L(\text{n-GMGR}_X)$ denote the language families defined by general n-MGN in the X mode and general n-MGR in the X mode, where $X = \{\text{union}, \text{conc}, \text{first}\}$. Then,

$$L(\text{MG}) = L(\text{n-GMGR}_X) = L(\text{n-GMGN}_X)$$

Illustration:

